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Tachyon condensation on brane sphalerons

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Abstract: We consider a sphaleron solution in field theory that provides a toy model for unstable D-branes of string theory. We investigate the tachyon condensation on a Dp-brane. The localized modes, including a tachyon, arise in the spectrum of a sphaleron solution of a ϕ^4 field theory on $\mathbb{M}^{p+1} \times S^1$. We use these modes to find a multiscalar tachyon potential living on the sphaleron world-volume. A complete cancelation between brane tension and the minimum of the tachyon potential is found as the size of the circle becomes small.

Keywords: Field Theories in Higher Dimensions, Tachyon Condensation.

Contents

1. Introduction

The existence of tachyon modes living on a D-brane anti-D-brane pair [1, 2] or on a non-BPS D-brane [3 – 9] of type IIA or IIB string theory is associated with the spectrum of open strings ending on D-branes. Several general arguments [3, 10 – 15] assert the tachyon potential has a minimum that represents the closed string vacuum without D-branes. In order for this to be true the negative energy density given by the tachyon potential at the minimum must exactly cancel the sum of the tensions of a D-brane anti-D-brane pair or the tension of a non-BPS D-brane (Sen's conjecture). The evidence of this conjecture by using string field theory [16] has been well explored in the literature [17 – 30]. Following the level truncation scheme initiated in Ref. [31], specially in refs. [32, 33], the evidence of this conjecture was first investigated in open bosonic string field theory [32] and in open superstring field theory [33], where the vacuum of the tachyon potential cancels about 99% and 85% of the D-brane tension, respectively. This is a phenomenon of tachyon condensation whose analysis of course requires computation of the tachyon potential. For earlier developments on tachyon condensation see refs. [34]. While such computation in string theory is complicated, in field theory the computation of the tachyon potential is much simpler. Thus, a way of investigating the tachyon condensation involving analytical solutions is the use of toy models in field theory. An interesting study on this direction can be found in refs. [35, 36].

In this paper, following the lines of [35], we present another example where we can easily investigate the tachyon condensation. We study a field theory presenting a sphaleron solution whose discrete spectrum has a tachyon mode, the zero mode and other massive modes. We integrated out these modes over a compact coordinate to find an effective action that we identify as describing the world-volume of a non-BPS Dp-brane of string field theory. We focus our attention on the multiscalar tachyon potential living on this world-volume to study tachyon condensation at several levels.

Sphaleron solutions are static and unstable classical solutions, localized in space, whose existence is associated with non-contractible loops in the space of field configurations [37, 38]. In string theory unstable D-branes can be understood as analogs of sphalerons of field theory, and can be referred to as D-brane sphalerons or simply "D-sphalerons" [9]. Furthermore the AdS/CFT correspondence has been used to show that unstable $D0$-brane in type IIB on AdS_5 has a field theory dual which is a sphaleron in gauge theories on $S^3\times\mathbb{R}$ [39] — see also [40] for recent developments. Following the perspective of ref. [39], one could investigate other field theoretical models, exhibiting sphalerons, being corresponding holographic of strings moving on a higher dimensional special manifold $\mathcal{M}$, just as in the AdS/CFT correspondence. For simplicity, in this paper, we focus only on the study of sphaleron solution of a ϕ^4 theory on $\mathbb{M}^{p+1}\times S^1$, where $\mathbb{M}^{p+1}$ is a $p+1$ dimensional Minkowski space. One can regard this field theory in $\mathbb{M}^{p+1}\times S^1$ as a corresponding holographic of strings moving on a special manifold $\mathcal{M}_{p+3}$. In this picture, one may find that sphalerons on the field theory side are dual to D-sphalerons [9] on the string theory side. This ϕ^4 theory, up to a constant, is the open superstring field [33, 41] restricted to the tachyon only [35, 36]. The sphaleron solution that we are dealing with is a static unstable periodic solution given in terms of Jacobi elliptic function. As one knows this solution is labeled by a real elliptic modulus parameter $0\leq k<1$ [43, 42]. We show that as k approaches zero the size of the circle S^1 becomes small and the cancelation between the negative energy density contribution from the tachyon potential and the tension of the non-BPS Dp-brane becomes more and more accurate at lower levels. In the limit $k\to 0$ the multiscalar tachyon potential we obtain is exact at level zero, being the extra scalars just spurious states. This tachyon potential has two minima that exactly cancels the tension of the non-BPS Dp-brane. This result in field theory mimics the expected result in string theory according to Sen's conjecture. Since the minima of the tachyon potential are invariant under Z_2 symmetry there exists tachyon kink (anti-kink) connecting these minima representing a D-brane with one lower dimension [44, 33, 45]. The paper is organized as follows. In Sec. 2 we obtain the spectrum of the sphaleron solution and the effective action. In section 3 we obtain the multiscalar tachyon potential and discuss the tachyon condensation. In section 4 we present our comments and conclusions.

2. Effective action for the sphaleron solution spectrum

In this section we consider a scalar field theory in $p+2$ dimensional space-time with the topology $\mathbb{M}^{p+1}\times S^1$. The action is

$$S=\int dt\,d^py\,dx\left[\frac{1}{2}\partial_M\Phi\,\partial^M\Phi-V(\Phi)\right],$$
$$=\int dy^{p+1}\,dx\left\{\frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial t}\right)^2-\left(\frac{\partial\Phi}{\partial y_i}\right)^2-\left(\frac{\partial\Phi}{\partial x}\right)^2\right]-V(\Phi)\right\}, \tag{2.1}$$

with $M=(y^\mu,x)$ and Φ is a real scalar field. $y^\mu=(t,y_i)$, $i=1,2,\ldots,p$, are coordinates on the world-volume of a Dp-brane embedded in a $p+2$ dimensional space-time with the coordinate x compactified on a circle S^1. This Dp-brane is represented by a static nontopological soliton solution $\bar{\Phi}$ of the equation of motion

$$\Box_{(p+2)}\Phi = -\frac{\partial V}{\partial \Phi}\,, \tag{2.2}$$

for $\bar{\Phi}$ depending only on the compact coordinate x. We take the positive semi-definite scalar potential of a Φ^4 theory

$$V(\Phi) = \frac{1}{2}\,(\Phi^2 - 1)^2\,. \tag{2.3}$$

This potential, up to a constant, can be viewed as the truncation of open superstring field theory [33, 41] restricted to the tachyon field only [35, 36]. In the background (2.1), this potential in the unstable vacuum $\Phi = 0$, i.e., before tachyon condensation, is the tension of the space-filling $(p+1)$-brane given by $T_{p+1} = V(\Phi = 0)$. Below we study the tachyon condensation from the point of view of a field theory living on unstable solution of this theory. This solution defines an unstable Dp-brane with tension T_p.

As is well-known the potential (2.3) does not produce lump solutions with tachyon mode on its fluctuation spectrum when the coordinate x has infinite size. However when x is compactified on a circle of length L [46 − 49] this potential does provide static periodic solutions $\bar{\Phi}(x) = \bar{\Phi}(x + L)$, with tachyon mode on its fluctuation spectrum. They are sphaleron solutions and can be expressed in terms of Jacobi elliptic function, i.e.,

$$\bar{\Phi}(x) = k\,b(k)\,\mathrm{sn}(b(k)\,x, k)\,, \qquad b(k) = \sqrt{\frac{2}{1+k^2}}, \tag{2.4}$$

where $0 \leq k < 1$ is a real elliptic modulus parameter. This is a static periodic function with period $4K(k)/b$, where $K(0) = \pi/2$ and $K(1) = \infty$. $K(k)$ is the complete elliptic integral of the first kind. Note that for a coordinate x with infinite size, the period is infinite ($k = 1$) and the solution becomes $\bar{\Phi}(x) = \tanh(x)$, which is a kink solution without any tachyon mode on its spectrum. Let us now observe the following. In conformal field theory, e.g. four dimensional Yang-Mills theory, there are no static finite energy solution (stable or unstable) because there is no scale to fix the mass of the solution. However sphalerons can be found if one considers the theory on $S^3 \times \mathbb{R}$ [39]. The size R of the sphere S^3 fixes the mass of any static solution as $\sim 1/R$. Similarly, in Φ^4 theory, the sphaleron solution can be found only if a coordinate is compactified on a circle S^1 with size $L = 4K(k)/b$. Furthermore, the mass of the tachyon is fixed as ${M_0}^2 \sim -1/R^2$, where one defines $R = L/2\pi$. In the limits discussed above, we readily find masses ${M_0}^2 \to -2$ ($k \to 0$) and ${M_0}^2 \to 0$ ($k \to 1$), as we can confirm latter by computing the sphaleron spectrum.

Let us now expand the action (2.1) around the sphaleron solution (2.4). We follow the lines of ref. [35] performed for lump solution. Consider the transformation $\Phi \to \bar{\Phi} + \eta$, $S(\Phi) \to S(\bar{\Phi} + \eta)$ into (2.1) such that

$$S = \int d^{p+1}y\,dx\Bigg[-\frac{1}{2}\left(\frac{d\bar{\Phi}}{dx}\right)^2 - V(\bar{\Phi}) - \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta + \frac{1}{2}\left(\eta\,\frac{\partial^2\eta}{\partial x^2} - \eta V''(\bar{\Phi})\,\eta\right) - \\ - \frac{V'''(\bar{\Phi})}{3!}\eta^3 - \frac{V''''(\bar{\Phi})}{4!}\eta^4\Bigg]\,, \tag{2.5}$$

(the primes mean derivatives with respect to argument of the function.) Since we are considering the Φ^4 theory (2.3), the expansion above is exact. We define above $y^\mu = (t, y_i)$ with a mostly plus signature $(-, +, +, \ldots, +)$. The first two terms of the expansion are related to the energy of the sphaleron solution or simply Dp-brane tension

$$T_p = \int_0^{4K/b} dx \left[\frac{1}{2} \left(\frac{d\bar{\Phi}}{dx} \right)^2 + V(\bar{\Phi}) \right] . \tag{2.6}$$

The fluctuations on the Dp-brane is governed by the quadratic η terms of (2.5). They provide a Schroedinger-like equation for the fluctuations η given as

$$-\frac{d^2\psi_n(x)}{dx^2} + V''(\bar{\Phi}(x))\,\psi_n(x) = {M_n}^2\,\psi_n(x)\,, \tag{2.7}$$

where we have considered the normal mode expansion

$$\eta(y^\mu; x) = \sum_n \xi_n(y)\psi_n(x)\,, \tag{2.8}$$

and the fact that the fields $\xi_n(y)$ living on the Dp-brane world-volume satisfy their equations of motion

$$\Box_{(p+1)}\,\xi_n(y) = {M_n}^2\xi_n(y)\,. \tag{2.9}$$

Let us now use (2.7), the scalar potential given in (2.3) and the sphaleron solution (2.4) to obtain its spectrum. The Schroedinger-like equation now reads

$$-\frac{d^2\psi_n(x)}{dx^2} + [6\,k^2\,b(k)^2\,\mathrm{sn}^2(b(k)\,x, k) - 2]\,\psi_n(x) = {M_n}^2\,\psi_n(x)\,. \tag{2.10}$$

This can be recognized as Lamé equation

$$-\frac{d^2\psi_n(z)}{dz^2} + N(N+1)\,k^2\,\mathrm{sn}^2(z, k)\,\psi_n(z) = h_n\,\psi_n(z)\,, \tag{2.11}$$

with $z = b(k)\,x$ and $h_n = ({M_n}^2 + 2)/\,b(k)^2$. The spectrum of this quantum mechanics problem is well-known [43, 42]. The spectrum concerns $2N + 1$ discrete states (for N positive integer) that are edges of N bound bands followed by a continuum band — see also [50, 51]. We shall focus only on the band edges, i.e., the discrete states. This will be further justified as we discuss the level expansion in section 3. In the case we are considering above $N = 2$, so that we have five discrete states with eigenfunctions given in terms of Jacobi elliptic functions and respective eigenvalues

$$\psi_0 = \mathrm{sn}^2(z, k) - \frac{1 + k^2 + \sqrt{1 - k^2 + k^4}}{3k^2}\,, \quad {M_0}^2 = (1 + k^2 - 2\sqrt{1 - k^2 + k^4})\,b(k)^2\,, \tag{2.12}$$

$$\psi_1 = \mathrm{cn}(z, k)\,\mathrm{dn}(z, k)\,, \quad {M_1}^2 = 0\,, \tag{2.13}$$

$$\psi_2 = \mathrm{sn}(z, k)\,\mathrm{dn}(z, k) \quad {M_2}^2 = 3\,k^2\,b(k)^2\,, \tag{2.14}$$

$$\psi_3 = \mathrm{sn}(z, k)\,\mathrm{cn}(z, k) \quad {M_3}^2 = 3\,b(k)^2\,, \tag{2.15}$$

$$\psi_4 = \mathrm{sn}^2(z, k) - \frac{1 + k^2 - \sqrt{1 - k^2 + k^4}}{3k^2}\,, \quad {M_4}^2 = (1 + k^2 + 2\sqrt{1 - k^2 + k^4})\,b(k)^2\,. \tag{2.16}$$

Note that in the interval $0 \leq k < 1$ the mass ${M_0}^2 < 0$, which implies that ψ_0 is always a tachyon mode. We have the zero mode ψ_1 and the remaining are all massive modes. Now we are ready to compute the action of the Dp-brane world-volume. Let us restrict ourselves to discrete modes only, i.e.,

$$\eta(y^\mu; x) = \xi_0(y)\psi_0(x) + \xi_1(y)\psi_1(x) + \xi_2(y)\psi_2(x) + \xi_3(y)\psi_3(x) + \xi_4(y)\psi_4(x)\,. \qquad (2.17)$$

We substitute this mode expansion into eq. (2.5) and integrate out all the modes over a period $4K(k)/b$. We have normalized the eigenfunctions in this period and used the orthonormality condition

$$\int_0^{4K/b} dx\, \psi_n(x)\, \psi_m(x) = \delta_{m,n}\,. \qquad (2.18)$$

This gives rise to a field theory living on the world-volume of the sphaleron representing a non-BPS Dp-brane with action

$$S_p = \int d^{p+1}y \left\{ -T_p - \frac{1}{2}\sum_{n=0}^{4} \partial_\mu\, \xi_n(y)\, \partial^\mu\, \xi_n(y) - V(\xi) \right\}. \qquad (2.19)$$

This is a theory of five real scalar fields in $p+1$ dimensional space-time that we get from a Φ^4 theory living in $p+2$ dimensions as we compactify one extra spatial dimension. The term T_p is the tension of the Dp-brane given by (2.6). In the tachyon condensation, this term must exactly cancel the minima of the tachyon potential at critical points ξ^*, i.e.,

$$T_p + V(\xi^*) = 0\,. \qquad (2.20)$$

At $\xi = \xi^*$ the non-BPS Dp-brane is indistinguishable from the vacuum where there is no D-branes. This is analogous to Sen's conjecture in string theory [29].

One can also translate the action (2.19) to the usual DBI like action for the tachyon field dynamics on the world-volume of a Dp-brane in string theory [52, 53], i.e.,

$$S_p = -\int d^{p+1}y\, V(T)\sqrt{1 + \partial_\mu T \partial^\mu T}\,. \qquad (2.21)$$

Let us consider the action (2.19) truncated up to the tachyon field ξ_0, and assume the following procedure [54, 45]:

$$V(\xi_0) = \left[\frac{\partial \xi_0(y)}{\partial T}\right]^2 - T_p = V(T) - T_p\,, \qquad (2.22)$$

$$\partial_\mu \xi_0(T(y)) = \frac{\partial \xi_0(y)}{\partial T}\, \partial_\mu T(\xi_0(y))\,. \qquad (2.23)$$

Now upon such considerations we can write

$$S_p = \int d^{p+1}y \left\{ -T_p - \frac{1}{2}\partial_\mu\, \xi_0(y)\, \partial^\mu\, \xi_0(y) - V(\xi) \right\} \qquad (2.24)$$

$$= \int d^{p+1}y \left\{ -V(T) - \frac{1}{2}V(T)\, \partial_\mu T \partial^\mu T \right\}. \qquad (2.25)$$

This is precisely the action (2.21) expanded up to quadratic first derivative terms. Such an approximation is good as long as higher order derivatives of T are small. This is indeed the case for tachyon matter [53]. Finally, the inclusion of the other scalar fields becomes straightforward as we use the same procedure.

3. The multiscalar tachyon potential and tachyon condensation

As we mentioned above we should integrate out all the modes over the compact coordinate x into the action (2.5) in order to find the effective action of the modes living on the world-volume of the sphaleron (2.19). The multiscalar tachyon potential living on such world-volume reads

$$V(\xi) = \int_0^{4K/b} dx \left\{ \frac{1}{2}\left(\eta \frac{\partial^2 \eta}{\partial x^2} - \eta V''(\bar{\Phi})\, \eta \right) - \frac{V'''(\bar{\Phi})}{3!}\eta^3 - \frac{V''''(\bar{\Phi})}{4!}\eta^4 \right\}. \tag{3.1}$$

This is an integration involving the sphaleron solution $\bar{\Phi}$ and the normal modes ψ_n performed in a period $4K/b$. Since these objects are labeled by the parameter $0 \leq k < 1$, this means we have distinct multiscalar potentials for distinct values of k.

Let us now analyze the tachyon condensation in the level expansion. We test the validity of the equation (2.20), i.e., we investigate how much $V(\xi^*)$ approaches the tension T_p at each level. We do this for several values of k. As is usual we assign *level zero* to the tachyon field ξ_0. To any other field ξ_i ($i = 1,2,3,4$) we must assign the *level* $L_i = |{M_i}^2 - {M_0}^2|$, with masses ${M_i}^2$ given by equations (2.12)–(2.16). In the limit $k \to 0$, the level of each field becomes $L_1 \to 2$, $L_2 \to 2$, $L_3 \to 8$ and $L_4 \to 8$, while in the limit $k \to 1$ we find $L_1 \to 0$, $L_2 \to 3$, $L_3 \to 3$ and $L_4 \to 4$. This means that for small values of k the fields become strongly effective in contributing to the potential even in lower levels, while for $k \to 1$ the fields become weakly effective to make such contribution so that we need more and more fields, i.e., higher levels. In other words, the multiscalar tachyon potential at the critical points, i.e., $V(\xi^*)$ approaches the tension T_p as we add scalar fields. This approach happens very quickly for $k \to 0$ and very slowly for $k \to 1$. As a consequence, in the case $k \to 0$ few scalar fields are relevant while in the opposite case $k \to 1$ most of the scalar fields, including those from the continuum spectrum, become relevant. This is evident from the explicit calculations and figure 1 below. Thus it is reasonable to expect that in the regime $k \to 1$, the contributions to the potential coming from continuum states become important. These are states of the bound and continuum bands we mentioned earlier. For each $k < 1$ large enough, there are zones where the values of energy allowed form a continuum band (Brillouin zone). Below, however, we focus our attention on values of $k > 0$, but small enough.

We now first focus our attention on the multiscalar tachyon potential living on the sphaleron world-volume for $k^2 = 1/32$. The potential is given in terms of five scalar fields

$$\begin{aligned} V(\xi) = {} & -\frac{232}{255}\xi_0^2 + \frac{1}{11}\xi_2^2 + \frac{32}{11}\xi_3^2 + \frac{355}{122}\xi_4^2 + \frac{200}{1817}\xi_0^4 + \frac{194}{1167}\xi_1^4 + \frac{179}{1094}\xi_2^4 + \frac{63}{382}\xi_3^4 + \frac{145}{879}\xi_4^4 + \\ & + \frac{285}{302}\xi_1^2\xi_4\xi_0 - \frac{270}{293}\xi_4\xi_2^2\xi_0 - \frac{9}{1216}\xi_3^2\xi_4\xi_0 - \frac{150}{217}\xi_3\xi_1\xi_0 - \frac{170}{247}\xi_2\xi_4\xi_0 + \frac{47}{3176}\xi_0^3\xi_4 + \\ & + \frac{105}{163}\xi_2^2\xi_0^2 + \frac{177}{262}\xi_1^2\xi_0^2 + \frac{162}{325}\xi_4^2\xi_2 + \frac{42}{2675}\xi_2\xi_3\xi_4\xi_1 - \frac{334}{179}\xi_1\xi_3\xi_2\xi_0 - \frac{16}{2163}\xi_4^3\xi_0 + \\ & + \frac{188}{285}\xi_0^2\xi_3^2 + \frac{118}{179}\xi_0^2\xi_4^2 + \frac{191}{579}\xi_1^2\xi_2^2 + \frac{239}{363}\xi_3^2\xi_1^2 + \frac{173}{266}\xi_1^2\xi_4^2 + \frac{39}{59}\xi_3^2\xi 2^2 + \frac{220}{329}\xi_2^2\xi_4^2 + \\ & + \frac{223}{676}\xi_4^2\xi_3^2 + \frac{1161}{2423}\xi_0^2\xi_2 + \frac{208}{851}\xi_1^2\xi_2 + \frac{41}{5286}\xi_3\xi_4\xi_1 + \frac{166}{681}\xi_2^3 + \frac{239}{487}\,\xi_3^2\,\xi_2\,. \end{aligned} \tag{3.2}$$

As a first approximation let us consider the multiscalar tachyon potential at level zero where only the tachyon field ξ_0 is present, i.e.,

$$V_0 = -\frac{232}{255}\,\xi_0^2 + \frac{200}{1817}\,\xi_0^4\,. \tag{3.3}$$

The nontrivial critical points are $\xi_0^* \simeq \pm\,2.03293$ in which the potential assumes the absolute value $|V^0(\xi^*)| \simeq 1.88001$. The Dp-brane tension is $T_p \simeq 2.27068$, thus

$$\frac{|V^0(\xi^*)|}{T_p} \simeq 0.827950\,, \tag{3.4}$$

which corresponds to about 82.79% of the expected answer. Before going to the next level let us study the expectation value of the fields at zero energy of vacuum

$$\langle \xi_n \rangle = \int_0^{4K/b} dx\,\psi_n(x)\,(\Phi_0 - \bar{\Phi}(x))\,. \tag{3.5}$$

We define the critical point of the tachyon potential as [35]

$$\Phi_0 - \bar{\Phi}(x) = \xi_0(y)\psi_0(x) + \xi_1(y)\psi_1(x) + \xi_2(y)\psi_2(x) + \xi_3(y)\psi_3(x) + \xi_4(y)\psi_4(x)\,, \tag{3.6}$$

being $\Phi_0 = 1$, the vacuum of original Φ^4 theory (2.3). After computing (3.5) for $k^2 = 1/32$, we find that the expectation values $\langle\xi_1\rangle$ and $\langle\xi_3\rangle$ are zero. Thus these fields play no role in our analysis of tachyon condensation, such that they can be removed from the theory. These are analogs of twist odd states in string field theory [35]. The level of each remaining scalar field ξ_2 and ξ_4 is $L_2 \simeq 2$ and $L_4 \simeq 8$, respectively. So we can work out approximations (L, I) with fields up to level L involving interactions up to level I.

Let us first consider the approximation $(2, 8)$, where terms involving the field ξ_2 up to fourth power are allowed. In this approximation the multiscalar tachyon potential reads

$$V^{(2,8)} = -\frac{232}{255}\,\xi_0^2 + \frac{1}{11}\,\xi_2^2 + \frac{166}{681}\,\xi_2^3 + \frac{1161}{2423}\,\xi_0^2\,\xi_2 + \frac{105}{163}\,\xi_0^2\,\xi_2^2 + \frac{200}{1817}\,\xi_0^4 + \frac{179}{1094}\,\xi_2^4\,. \tag{3.7}$$

This potential has the nontrivial critical points $\xi_0^* \simeq \pm 2.12944$, $\xi_2^* \simeq -0.373448$, where $|V^{(2,8)}(\xi^*)| \simeq 2.26312$. At this level we find

$$\frac{|V^{(2,8)}(\xi^*)|}{T_p} \simeq 0.996671\,, \tag{3.8}$$

that corresponds to about 99.67% of the expected result. This nicely improve the result obtained at level zero. We can still proceed up to approximation (8,32), where we turn on the field ξ_4 to which we assign the level $L_4 \simeq 8$, and consider its interactions up to fourth power. Now the critical points of the potential are

$$\xi_0^* \simeq -2.13221,\ \xi_2^* \simeq -0.37193,\ \xi_4^* \simeq 0.03589\,. \tag{3.9}$$

They are in perfect agreement with expectation values $\langle\xi_0\rangle$, $\langle\xi_2\rangle$, and $\langle\xi_4\rangle$, as we can check by using (3.5). At these critical points $|V^{(8,32)}(\xi^*)| \simeq 2.27062$ and then

$$\frac{|V^{(8,32)}(\xi^*)|}{T_p} \simeq 0.999974\,. \tag{3.10}$$

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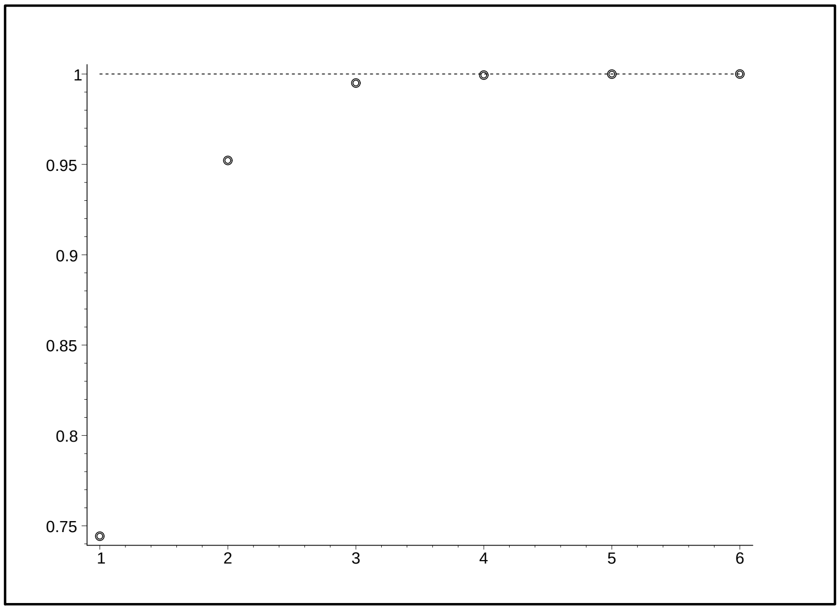

Figure 1: The ratio $|V(\xi^*)|/T_p = 0.744314,\ 0.952164,\ 0.994983,\ 0.999376,\ 0.999927$, and 0.999974 for the six values $k^2 = 3/4,\ 1/2,\ 1/4,\ 1/8,\ 1/16$ and $1/32$, respectively, at maximal approximation.

In this approximation the cancelation between the brane tension and the minimum of the tachyon potential corresponds to about 99.99% of the expected answer. Furthermore we have calculated the ratio $|V^{(L,I)}(\xi^*)|/T_p$ for several values of $0 \le k^2 < 1$ at maximal approximation (L, I). We found that this ratio approaches unity as $k \to 0$, as can be seen in figure 1.

Let us now consider explicit calculations for $k \to 0$. It should be understood that in this limit the sphaleron solution approaches $\bar{\Phi}(x) \simeq kb \sin bx$, whose quantum mechanics problem can still be treated via Lamé equation (2.11). The tachyon potential obtained in this case is given by

$$\begin{aligned} V(\xi) = {} & -\xi_0^2 + 3\,\xi_3^2 + 3\,\xi_4^2 + \frac{1}{4}\,\frac{\sqrt{2}\,\xi_0^4}{\pi} + \frac{3}{8}\,\frac{\sqrt{2}\,\xi_1^4}{\pi} + \frac{3}{8}\,\frac{\sqrt{2}\,\xi_2^4}{\pi} + \frac{3}{8}\,\frac{\sqrt{2}\,\xi_3^4}{\pi} + \frac{3}{8}\,\frac{\sqrt{2}\,\xi_4^4}{\pi} + \\ & + \frac{3}{2}\,\frac{\sqrt{2}\,\xi_4^2\,\xi_0^2}{\pi} + \frac{3}{4}\,\frac{\sqrt{2}\,\xi_4^2\,\xi_3^2}{\pi} + \frac{3}{2}\,\frac{\sqrt{2}\,\xi_3^2\,\xi_0^2}{\pi} + \frac{3}{2}\,\frac{\sqrt{2}\,\xi_0^2\,\xi_1^2}{\pi} - 6\,\frac{\xi_2\,\xi_1\,\xi_3\,\xi_0}{\pi} - 3\,\frac{\xi_2^2\,\xi_4\,\xi_0}{\pi} + \\ & + \frac{3}{2}\,\frac{\sqrt{2}\,\xi_2^2\,\xi_0^2}{\pi} + \frac{3}{2}\,\frac{\sqrt{2}\,\xi_2^2\,\xi_4^2}{\pi} + \frac{3}{2}\,\frac{\sqrt{2}\,\xi_4^2\,\xi_1^2}{\pi} + \frac{3}{2}\,\frac{\sqrt{2}\,\xi_2^2\,\xi_3^2}{\pi} + \\ & + \frac{3}{4}\,\frac{\sqrt{2}\,\xi_2^2\,\xi_1^2}{\pi} + \frac{3}{2}\,\frac{\sqrt{2}\,\xi_3^2\,\xi_1^2}{\pi} + 3\,\frac{\xi_4\,\xi_1^2\,\xi_0}{\pi}\,. \end{aligned} \tag{3.11}$$

Using (3.5) we find that the expectation values $\langle\xi_1\rangle$, $\langle\xi_2\rangle$, $\langle\xi_3\rangle$, and $\langle\xi_4\rangle$ are zero. This means that in the limit $k \to 0$ these fields are spurious states, such that we can remove all of them from the theory. Thus in this limit the relevant multiscalar tachyon potential simply reads

$$V(\xi) = -\xi_0^2 + \frac{1}{4}\frac{\sqrt{2}}{\pi}\,\xi_0^4\,, \tag{3.12}$$

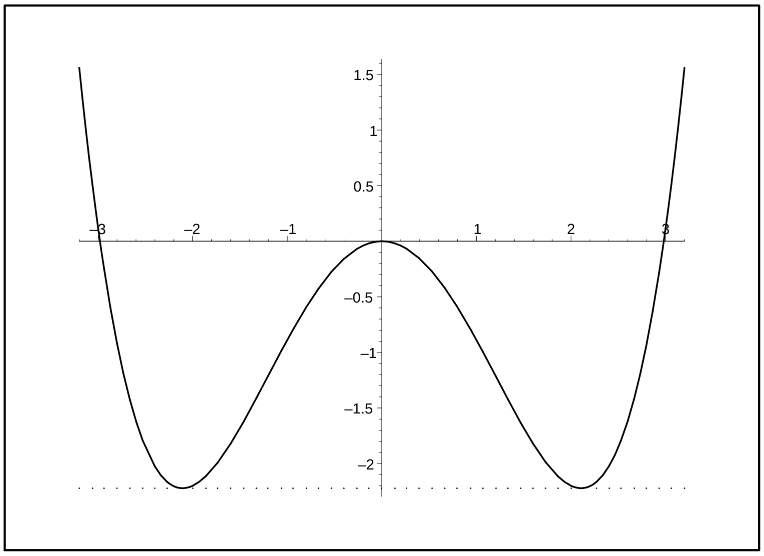

Figure 2: The tachyon potential at level zero as $k \to 0$. The dotted line stands for minus the brane tension T_p.

whose nontrivial critical points are $\xi_0^* = \pm\sqrt{\sqrt{2}\pi}$. At these points the potential assumes the minimal value $|V(\pm\sqrt{\sqrt{2}\pi})| = (1/2)\sqrt{2}\,\pi$. Here the study of tachyon condensation restricts only to level zero, i.e., we must consider just the tachyon field with no further scalars in the potential.

Since the Dp-brane tension in this case is given exactly by $T_p = (1/2)\sqrt{2}\,\pi$, thus

$$\frac{|V(\xi^*)|}{T_p} = 1\,, \tag{3.13}$$

which corresponds exactly to 100% of the expected answer ! (see eq. (2.20)) As we mentioned earlier in the limit $k \to 0$ few states are needed to contribute to the potential. Here we confirm that only the tachyon field ξ_0 effectively contributes to the potential even at level zero. The tachyon potential given by (3.12) behaves as depicted in figure 2. Note that this tachyon has the same mass, ${M_0}^2 = -2$, of the tachyon in the original Φ^4 theory given in (2.3), as it happens in string theory [35]. Note also because the tachyon potential is invariant under Z_2 symmetry, it supports a tachyon kink interpolating between the two minima of the potential at $\xi_0^* = \pm\sqrt{\sqrt{2}\pi}$. This kink represents a BPS D-brane of one lower dimension, i.e., a $D(p-1)$-brane, with tension T_{p-1} [6, 8, 44, 33, 45]. We can readily use the Dp-brane action (2.19) to obtain the kink solution $\xi_0(y)$ and its tension T_{p-1}

$$\xi_0 = \pm\xi_0^* \tanh(y)\,, \qquad T_{p-1} = \frac{4}{3}(\xi_0^*)^2\,. \tag{3.14}$$

4. Conclusions

We investigate the cancelation between the brane tension and the minimum of the tachyon potential living on the world-volume of a sphaleron solution representing a non-BPS Dp-brane. We consider a scalar field Φ^4 theory in $p+2$ dimensions with one spatial coordinate compactified on a circle. This field theory produces a sphaleron solution that is static and localized in space, but unstable. In the sphaleron spectrum there is a tachyon mode, the zero mode and other massive modes. We integrate out these modes over the compact coordinate to obtain the multiscalar tachyon potential living on the sphaleron world-volume. The compact coordinate has a size $L = 4K(k)/b$ depending on the elliptic modulus parameter $0 \leq k < 1$ of the sphaleron solution. The brane tension T_p is fixed for a given k, while the minimum of the potential $V(\xi^*)$ depends on the level of approximation. Thus the cancelation between them depends crucially on the efficiency of the modes in contributing to the minimum of the potential in the level expansion. In the limit $k \to 1$, the compact coordinate becomes large and the efficiency of each mode in contributing to the minimum of the potential also increases. On the other hand, in the limit $k \to 0$, this efficiency for higher modes decreases such that just the tachyon mode contributes to the minimum of the potential. In this case, we have shown there exists a perfect cancelation between T_p and $V(\xi^*)$, which completely agrees with expected answer (2.20). One could extend our analysis of tachyon condensation on sphalerons in field theory to investigations in string theory. These studies might shed some light on several issues such as T-duality and closed string tachyons.

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